

Math 180, Quiz 7 Retest: Tuesday, March 19, 2013

Take fifteen minutes to complete this quiz. Please show all your work, and write your name on the front and back of the paper before turning it in. Make sure to show all relevant work.

1. (4 pts) Find the absolute extrema of  $f(x) = \frac{1}{\sin(x) - 1}$  on the interval  $[-\pi, 0]$ .

$$f'(x) = \frac{-(-\cos(x))}{(\sin(x) - 1)^2} = \frac{\cos(x)}{(\sin(x) - 1)^2}$$

$\cos(x) = 0 \Rightarrow x = \dots, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots$   
 $\sin(x) = 1 \Rightarrow x = \dots, \frac{-3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \dots$   
 critical point:  $x = -\pi/2$

$f(0) = \frac{1}{\sin(0) - 1} = \frac{1}{0 - 1} = -1$   
 $f(-\pi/2) = \frac{1}{\sin(-\pi/2) - 1} = \frac{1}{-1 - 1} = -1/2$  (max)  
 $f(-\pi) = \frac{1}{\sin(-\pi) - 1} = \frac{1}{0 - 1} = -1$  (min)

2. (5 pts) Helium is pumped into a balloon at a constant rate of  $1 \text{ cm}^3/\text{s}$ . The balloon can only stand being stretched to a total surface area of  $5,000 \text{ cm}^2$ . How fast is the diameter of the balloon expanding right as the balloon pops? (You probably want to work with radii first and then convert to diameters at the end.)

$$V = \frac{4}{3}\pi r^3 \quad A = 4\pi r^2 = 5000 \text{ cm}^2 \quad V' = 1 \text{ cm}^3/\text{s}$$

$$V' = 4\pi r^2 r'$$

$$\Rightarrow r' = \frac{V'}{4\pi r^2} = \frac{1 \text{ cm}^3/\text{s}}{5000 \text{ cm}^2} = \frac{1}{5000} \text{ cm/s}$$

$$D = 2r \Rightarrow D' = 2r'$$

$$D' = \frac{1}{2500} \text{ cm/s}$$

3. (1 pt) Suppose  $f$  and  $f^{-1}$  are differentiable functions, and that you are given the values

$x$	$f(x)$	$f'(x)$
0	3/2	1/2
1	2	1/2
3/2	19/8	1
2	3	3/2
5/2	31/8	2
3	5	5/2

What is  $(f^{-1})'(3)$ ? (Show at least enough work that I can tell you didn't just make a lucky guess!)

Recall:

$$f(f^{-1}(x)) = x \quad f^{-1}(3) = 2 \quad f'(2) = \frac{3}{2}$$

$$f'(f^{-1}(x)) (f^{-1})'(x) = 1$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} \quad (f^{-1})'(3) = \frac{1}{3/2} = \frac{2}{3}$$

$$\frac{2}{3}$$